1. **Sol.** A = {[3, 5, 9}

2. **Sol.** -

3. **Sol.**

We have

$f(x) = | x - 3 |$

Clearly, $f(x)$ is defined for all $x \in$ R

∴ Domain (f) = R.

Now,

$| x - 3 | \geq 0$ for all $x \in$ R

$\Rightarrow 0 \leq | x - 3 | < \infty$ for all $x \in$ R

$\Rightarrow 0 \leq f(x) < \infty$ for all $x \in$ R

$\Rightarrow f(x) \in [0, \infty)$ for all $x \in$ R

Hence, Range (f) = $[0, \infty)$

4. **Sol.** 324°

5. **Sol.**

LHS = cos (90° + 15°) + cos (90° - 75°)

= - sin 15° + sin 75° = RHs

6. **Sol.** -

7. **Sol.**

Let B = {b}. Then A = {a, B}.

∴ P(A) = {Φ, {a}, {B}, {a, B}}

= {Φ, {a}, {{b}}, {a, {b}}}.

8. **Sol.** A = {x, y, z}, B = {1, 2}.

9. **Sol.**

We have,

$$f(x) = \frac{x-2}{3-x}$$

Domain of f: Clearly, $f(x)$ is defined for all x satisfying $3 - x \neq 0$ i.e. $x \neq 3$.

Hence, Domain (f) = R – {3}.

Range of f: let $y = f(x)$, i.e.

$$y = \frac{x-2}{3-x}$$

$\Rightarrow 3y - xy = x - 2$

$\Rightarrow x(y + 1) = 3y + 2$

$$\Rightarrow x = \frac{3y+2}{y+1}$$

Clearly, x assumes real value for all y except $y + 1 = 0$ i.e. $y = -1$.

Hence, Range (f) = R – {-1}.

10. **Sol.**

We have,

$$\frac{\sin A}{\sin B} = p \text{ and } \frac{\cos A}{\cos B} = q$$

$$\Rightarrow \frac{\sin A}{\sin B} \cdot \frac{\cos B}{\cos A} = \frac{p}{q}$$

$$\Rightarrow \frac{\tan A}{\tan B} = \frac{p}{q}$$

$$\Rightarrow \frac{\tan A}{p} = \frac{\tan B}{q} = \lambda \text{ (say)}$$

$\Rightarrow$ tan A = p λ and tan B = q λ. ... (i)

Now, sin A = p sin B

$$\Rightarrow \frac{\tan A}{\sqrt{1+\tan^2 A}} = p\frac{\tan B}{\sqrt{1+\tan^2 B}}$$

$$\Rightarrow \frac{p\lambda}{\sqrt{1+p^2\lambda^2}} = p \cdot \frac{q\lambda}{\sqrt{1+q^2\lambda^2}}$$

$$\Rightarrow p^2(1+q^2\lambda^2) = p^2q^2(1+p^2\lambda^2)$$

$$\Rightarrow \lambda^2(q^2 - p^2q^2) = q^2 - 1$$

$$\Rightarrow \lambda^2 = \frac{q^2-1}{q^2(1-p^2)}$$

$$\lambda = \pm\frac{1}{q}\frac{\sqrt{q^2-1}}{1-p^2}$$

$$\therefore \tan A = \pm\frac{p}{q}\frac{\sqrt{q^2-1}}{1-p^2} \text{ and } \tan B = \pm\frac{\sqrt{q^2-1}}{1-p^2}$$

11. **Sol.**

We have,

$$LHS = \frac{\cos(90^\circ+\theta)\sec(-\theta)\tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\sin(180^\circ+\theta)\cos(90^\circ-\theta)}$$

$$\Rightarrow LHS = \frac{(-\sin\theta)(\sec\theta)(-\tan\theta)}{(\sec\theta)(-\sin\theta)(\tan\theta)} = -1 = RHS$$

12. **Sol.**

We have,

$$\sec\theta = \sqrt{2} \Rightarrow \cos\theta\frac{1}{\sqrt{2}}$$

$$\therefore \sin\theta = \pm\sqrt{1-\cos^2\theta} = \pm\sqrt{1-\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$$

But, θ lies in the third quadrant in which sin θ is negative.

$$\therefore \sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \text{cosec}\theta = -\sqrt{2}$$

And,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = -\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{1} = -1 \Rightarrow \cos = -1$$

$$\therefore \frac{1+\tan\theta+\cos ec\theta}{1+\cot\theta-\cos ec\theta}=\frac{1-1-\sqrt{2}}{1-1+\sqrt{2}}=-1$$

13. Sol.

We have,

$m^2+m'^2+2mm'\cos\theta=1$ and

$n^2+n'^2+2nn'\cos\theta=1$

$\Rightarrow m'^2+2mm'\cos\theta+m^2\cos^2\theta-m^2\cos^2\theta+m^2=1$

And,

$n'^2+2nn'\cos\theta+n^2\cos^2\theta-n^2\cos\theta+n^2=1$

$\Rightarrow (m'+m\cos\theta)^2+m^2(1-\cos\theta)=1 and (n'+n\cos\theta)^2+n^2(1-\cos^2\theta)=1$

$\Rightarrow (m'+m\cos\theta)^2=1-m^2\sin^2\theta$

And, $(n'+n\cos\theta)^2=1-n^2\sin^2\theta$

Now, $(m'+m\cos\theta)(n'+n\cos\theta)$

$=m'n'+(mn'+m'n)\cos\theta+mn\cos^2\theta$

$=-mn+mn\cos^2\theta$

$[\because mn+m'n'+(mn'+m'n)\cos\theta=0]$

$=-mn(1-\cos^2\theta)=-mn\sin^2\theta$

$\therefore (m'+m\cos\theta)^2(n'+n\cos\theta)^2=m^2n^2\sin^4\theta$

[On squaring both sides]

$\Rightarrow (1-m^2\sin^2\theta)(1-n^2\sin^2\theta)=m^2n^2\sin^4\theta$

[Using (i) and (ii)]

$\Rightarrow 1-(m^2+n^2)\sin^2\theta+m^2n^2\sin^4\theta=m^2n^2\sin^4\theta$

$\Rightarrow 1=(m^2+n^2)\sin^2\theta$

$\Rightarrow m^2+n^2=\cos ec^2\theta$

14. Sol.

(i) $f+g:(-\infty,1)\to R$ defined by

$(f+g)(x)=\log_e(1-x)+[x]$

(ii) $fg:(-\infty,1)\to R$ defined by

$(fg)(x)=[x]\log_e(1-x)$

(iii) $\frac{f}{g}:(-\infty,0)\to R$ defined by

$\left(\frac{f}{g}\right)(x)=\frac{\log_e(1-x)}{[x]}$

(iv) $\frac{g}{f}:(-\infty,0)\cup(0,1)\to R$ defined by

$\left(\frac{g}{f}\right)(x)=\frac{[x]}{\log_e(1-x)}$

$(f+g)(-1)=\log_e 2-1$

$(fg)(0)=0,\left(\frac{f}{g}\right)\left(\frac{1}{2}\right)$ does not exist.

$\left(\frac{g}{f}\right)\left(\frac{1}{2}\right)=0$

15. Sol.

We have,

$f(x)=x+1/x$

$\therefore f(x^3)=x^3+\frac{1}{x^3} and [f(x)]^3=\left(x+\frac{1}{x}\right)^3$

Now,

$[f(x)^3]=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=f(x^3)+3f(x) and f(x)=f\left(\frac{1}{x}\right)$

$\therefore [f(x)]^3=f(x^3)+3f\left(\frac{1}{x}\right)$

16. Sol.

Let P, Q and R be the sets of families buying newspaper A, B and C respectively. Let u be the universal set. Then,

n (P) = 40% of 10,000 = 4000, n (Q) = 20% of 10,000 = 2000,

n (R) = 10% of 10,000 = 1000, n (P ∩ Q) = 5 % of 10,000 = 500,

n (Q ∩ R) = 3% of 10,000 = 300, n (R ∩ P) = 4% of 10,000 = 400

n (P ∩ Q ∩ R) = 2% of 10,000 = 200 and $n(u)$ = 10,000.

(i) Required number = n (P ∩ Q’ ∩ R’) = n (P ∩ (Q ∪ R)’)

= n (P) – [P ∩ (Q ∩ R)]

[∵n (A ∩ B’) = n(A) – n (A ∩ B)]

= n (P) – n [(P ∩ Q) ∪ (P ∩ R)]

$= n(P) - [n(P \cap Q) + n(P \cap R) - n\{(P \cap Q) \cap (P \cap R)\}]$

$= n(P) - [n(P \cap R) + n(P \cap R) - n(P \cap Q \cap R)]$

$= 4000 - [500 + 400 - 200] = 3300$

(ii) Required number = $n(P' \cap Q \cap R') = n(Q \cap P' \cap R')$

$= n(Q \cap (P \cup R'))$

$= n(Q) - n(Q \cap (P \cup R))$

$[\because n(A \cap B') = n(A) - n(A \cap B)]$

$= n(Q) - n[(Q \cap P) \cup (Q \cap R)]$

$= n(Q) - [n(Q \cap P) + n(Q \cap R) - n\{(Q \cap P) \cap (Q \cap R)\}]$

$= n(Q) - [n(P \cap Q) + n(Q \cap R) - n(P \cap Q \cap R)]$

$= 2000 - [500 + 300 - 200] = 1400$

(iii) Required number = $n(P' \cap Q' \cap' R) = n[(P \cup Q \cup R)']$

$= n(u) - n(P \cup Q \cup R)$

$= n(u) - [n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(R \cap P) + n(P \cap Q \cap R)]$

$= 10000 - [4000 + 2000 + 1000 - 500 - 300 - 400 + 200] = 4000$

17. Sol. We have,

$$f(x) = \log_4\left\{\log_5\left(\log_3(18x - x^2 - 77)\right)\right\}$$

Since $\log_a x$ is defined for all $x > 0$. Therefore, $f(x)$ is defined if

$$\log_5\{\log_3(18x - x^2 - 77)\} > 0 \text{ and } 18x - x^2 - 77 > 0$$

$\Rightarrow \log_3(18x - x^2 - 77) > 5^0$ and $x^2 - 18x + 77 > 0$

$\Rightarrow \log_3(18x - x^2 - 77) > 1$ and $(x - 11)(x - 7) < 0$

$\Rightarrow 18x - x^2 - 77 > 3^1$ and $7 < x < 11$

$\Rightarrow 18x - x^2 - 80 > 0$ and $7 < x < 11$

$\Rightarrow x^2 - 18x + 80 < 0$ and $7 < x < 11$

$\Rightarrow (x - 10)(x - 8) < 0$ and $7 < x < 11$

$\Rightarrow 8 < x < 10$ and $7 < x < 11$

$\Rightarrow 8 < x < 10$

$\Rightarrow x \in (8, 10)$.

Hence, the domain of $f(x)$ is $(8, 10)$.

18. Sol. -