1. Acceleration is 1 : 1.

Change in momentum $\Delta p = 2mv\cos 45^\circ$, where 45° is the angle with the normal.

$\Delta p = 2 \times 5 \times v \cos 45^\circ = \frac{10}{\sqrt{2}} v.$

2.

Since density is same $g = \frac{G\frac{4}{3}\pi r^3 \rho}{r^2}$, *i.e.*, $g \propto r$.

$\therefore \frac{g_1}{g_2} = \frac{r_1}{r_2}.$

3. In accelerated frames of reference

4.

Escape velocity $= \sqrt{\frac{2GM}{R}} = 11.2$ km/sec.

$\frac{\Delta r}{r} \times 100 = 2$

and $\frac{\Delta v}{v} = 3 \times \frac{\Delta r}{r}$ $\therefore \frac{\Delta v}{v} \times 100 = 3 \times 2 = 6.$

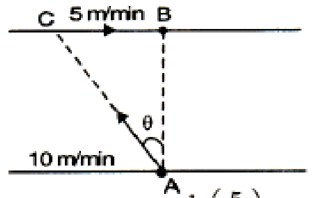
5. Velocity at the lowermost point should be

$v = \sqrt{5gl}$

6.

7.

To cross the river by the shortest path he should swim at an angle θ with the straight



line AB, *i.e.*, $\theta = \sin^{-1}\left(\frac{5}{10}\right) = 30^\circ$. Velocity component in the direction of shortest path, AB is,

$v = 10 \cos 30^\circ = \frac{10\sqrt{3}}{2} = 5\sqrt{3}$ m/min.

Time taken to cross the river

$$t = \frac{AB}{v \text{ along } AB} = \frac{1000}{5\sqrt{3}}$$

$$t = \frac{200}{\sqrt{3}} \text{ minutes}$$

8.

$l = \frac{1}{2}$ m, $m = 1$ kg, $v = 5$ m/sec (a constant).

Tension is maximum at the lower-most point.

$$T_{max} = mg + \frac{mv^2}{r} = g + \frac{25 \times 2}{1}$$

$= 60$ N. ...($m = 1$kg)

Tension is minimum at the top-most point

$$T_{min} = \frac{mv^2}{r} - mg = 50 - g = 40 \text{ N}.$$

9.

Time taken for the sound to be received =

$$T = \sqrt{\frac{2h}{g}} + \frac{h}{v}.$$

$T = 4.2$, $h = 78.4$ m, $v = ?$

$$4.2 = \sqrt{\frac{2 \times 78.4}{9.8}} + \frac{h}{v}$$

$$\Rightarrow \frac{h}{v} = 4.2 - 4 = 0.2$$

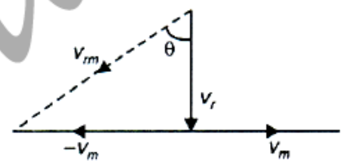
$$v = \frac{h}{0.2} = \frac{78.4}{0.2} = 392 \text{ m/sec.}$$

10. No, Acceleration of $\frac{v^2}{r}$ always point towards the centre and so the direction changes.

11. The man should hold his umbrella in the direction, that he feels the rain is coming, *i.e.*, V_{rm}.

We know, $\vec{V}_{rm} = \vec{V}_r - \vec{V}_m$

$\vec{V}_{rm} = \vec{V}_r + (-\vec{V}_m)$



$\vec{V}_{rm}$ as indicated in diagram makes an angle θ with the rain.

So the umbrella will be kept at an angle θ

$$= \tan^{-1}\left(\frac{V_m}{V_r}\right).$$

12.

We know, $A = lb$

Given : $l = 15.12$, $b = 10.15$,

$\Delta l = 0.01$, $\Delta b = 0.01$

$$= \frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

$\therefore$ % error in area $x = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) \times 100$

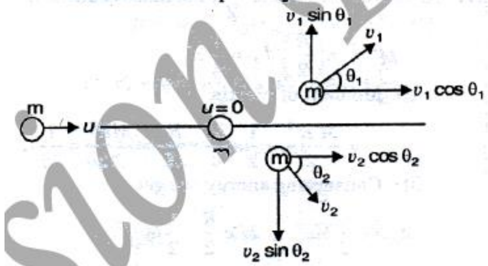
$$x = 100\left(\frac{0.01}{15.12} + \frac{0.01}{10.15}\right)$$

$$= 1\left(\frac{1}{15.12} + \frac{1}{10.15}\right)$$

$$x = \frac{25.27}{153.47} = 0.1646.$$

13.

Consider the velocity of the mass m under motion be v. As the collision is elastic and oblique, they will move at angles say θ_1 and θ_2 with velocities v_1 and v_2.



$$mv = mv_1 \cos\theta_1 + mv_2 \cos\theta_2$$

$$0 = mv_1 \sin\theta_1 - mv_2 \sin\theta_2$$

Squaring, adding and simplifying, we get

$$v^2 = v_1^2 + v_2^2 + 2v_1v_2(\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2)$$

$$= v_1^2 + v_2^2 + 2v_1v_2[\cos(\theta_1 + \theta_2)]$$

From conservation of energy, we get

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \Rightarrow v^2 = v_1^2 + v_2^2$$

$\therefore 2v_1v_2\cos(\theta_1 + \theta_2) = 0$

or $\theta_1 + \theta_2 = \frac{\pi}{2}$ or 90°

The angle between the line of their motion is 90°.

14.

Escape velocity: The minimum velocity required to escape from the gravitational pull of the earth is called escape velocity.

$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = 11.2$ km/sec.

Orbital velocity. The velocity with which a satellite orbits round the planet (earth) after being placed in it, is called orbital velocity. Gravitational force has to provide the necessary orbital velocity.

It is given by, $v_0 = \sqrt{\frac{GM}{R+h}}$, where

h is the height of the orbit from the surface of earth.

R is the radius and M is the mass of the earth.

In an orbit of radius r, a satellite of mass m moves round a planet of mass M. Then,

$$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

$$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}}$$

Where h is the height at which the satellite is from the surface of earth.

For close to earth orbits, $h = 0$

$$\therefore \quad v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

Since, $v_e = \sqrt{2gR}$, $v_0 = \frac{v_e}{\sqrt{2}}$

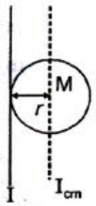
15.

(*i*) **Parallel Axis theorem.** The Moment of inertia I about an axis parallel to the axis through COM, is the sum of the moment of inertia about the axis through COM (I_{cm}) and the product of the total mass M and the perpendicular separation a between the axes for any rigid body.

(*ii*) $I = I_{cm} + Mr^2$

$$= \frac{Mr^2}{2} + Mr^2$$

$$= \frac{3}{2}Mr^2.$$



16.

(*i*) Acceleration should be uniform or a = costant.

(*ii*) We know that acceleration is the rate of change velocity and is $a = \frac{dv}{dt}$

$$\therefore \quad a = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{dv}{dx}.v$$

$$\therefore \quad adx = vdv$$

Integrating, we get

$$\int_{x_i}^{x_f} adx = \int_u^v vdv$$

$$a.x\Big|_{x_i}^{x_f} = \frac{v^2}{2}\Big|_u^v$$

$$a(x_f - x_i) = \frac{v^2 - x^2}{2}$$

$$\Rightarrow \quad v^2 - u^2 = 2a\,(x_f - x_i) = 2as.$$

(*iii*) Slope of $v - t$ graph gives acceleration

Area under $v - t$ graph gives displacement in a straight line.

OR

(*i*) The maximum value of static friction, preventing the body from sliding is called limiting friction.

(*ii*) Yes, friction is a self-adjusting force. Once the applied force is less than the limiting friction, the friction force experienced is the applied force.

(*iii*) Nature of the surface, and Noarmal reaction affect the frictional force.

(*iv*) The normal reaction on the vertical wall is N = F. The frictional force μN should balance the weight Mg i.e., $\mu N = Mg$

$$\therefore \quad \mu = \frac{Mg}{N} = \frac{Mg}{F}$$

17.

(*i*) Elastic Collision is one in which both momentum and energy are conserved. Inelastic collision is one in which only the momentum is conserved.

(*ii*) In elastic collision, we know, both K.E. and momentum will be conserved.

$$\therefore \frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

and $\quad m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

They can be written as,

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad ...(i)$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad ...(ii)$$

Dividing (*i*) and (*ii*), we have

$$u_1 + v_1 = v_2 + u_2$$

or

$$v_2 - v_1 = -(u_1 - u_2) = -(u_1 - u_2).$$

using $v_2 = v_1 - (u_1 - u_2)$ and the momentum conservation one can prove.

$$v_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{(m_1 + m_2)}$$

$$v_2 = \frac{(m_2 - m_1)u_2 + 2m_1u_1}{(m_1 + m_2)}$$

(*i*) If $m_1 >> m_2$ and $u_2 = 0$. We get

$$v_1 = \frac{m_1u_1}{m_1} = u_1, v_2 = 24,$$

(*ii*) If $m_1 >> m_2$ and $u_1 = 0$. We get

$$v_1 = \frac{2m_2u_2}{m_1}, \text{ as } m_2 << m_1 \therefore v_1 \approx 0$$

$$v_2 = -\frac{m_1u_2}{m_1} = -u_2$$

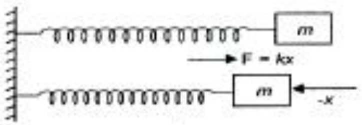
OR

(*i*) Consider a particle of mass m attached to a spring of spring constant, k. When spring is compressed through small distance x then restoring force comes into play

$$F = kx \quad ...(i)$$

This force brings the body back to its mean position and hence it starts oscillating. If $\frac{d^2x}{dt^2}$ is accleration produced,

$$F = \frac{md^2x}{dt^2} \quad ...(i)$$



From (i) & (ii) $\frac{md^2x}{dt^2} = -kx$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = \frac{-k}{m}x \Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \left(\omega^2 = \frac{k}{m}\right)$$

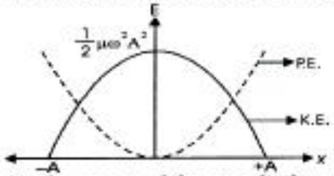
Now potential energy $U = \frac{1}{2} m\omega^2 x^2$.

Putting value of ω^2, we have $U = \frac{1}{2}kx^2$

(i) As position x changes from mean position 'O' to either extreme position '± A' on either side, the P.E increases causing a drop in K.E of the attached mass. The entire P.E of the stretched spring at the either extreme position is converted into K.E at mean position or vice-versa.

Thus law of conservation of energy is obeyed.

(ii)



(iii) Displacement of the attached mass from the mean position is given by

$$y = a \sin \omega t$$

$$\therefore \quad \text{velocity } v = \frac{dy}{dt} = a\,\omega \cos \omega t$$

$$= a\omega\sqrt{1-\sin^2 \omega t}$$

$$= a\omega\left(1-\frac{y^2}{a^2}\right)^{1/2}$$

$$= \omega\sqrt{a^2-y^2}$$

but $\quad \omega = \sqrt{\frac{k}{m}}$

$$\therefore \quad V = \sqrt{\frac{k}{m}} \cdot \sqrt{a^2-y^2}$$

$$= \left[\frac{k}{m}(a^2-y^2)\right]^{1/2}$$

18.

A body projected upward and allowed to fall due to gravity is called a projectile.

Consider a projectile having velocity v and angle of projection θ. The horizontal and vertical components are $v \cos \theta$ and $v \sin \theta$. Let P(x, y) be the point where the projectile will be in time 't'. Then,

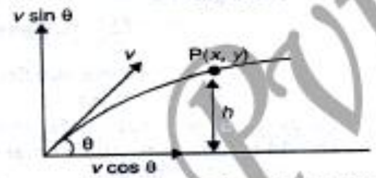
$$x = v \cos \theta\, t \quad \because a_x = 0$$

$$y = v \sin \theta\, t - \frac{1}{2}gt^2 \quad \because a_y = g \downarrow$$

$$\therefore \quad y = v \sin\theta \frac{x}{v\cos\theta} - \frac{1}{2}g\frac{x^2}{v^2\cos^2\theta}$$

$$= ax - bx^2$$

$$= x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta} \quad ...(i)$$



Since $y \propto x^2$, the path of a projectile is a parabola.

When the body returns to the same horizontal level, $y = 0$

$$\therefore \quad 0 = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

[From (i)]

$$\text{or} \quad x\tan\theta = \frac{gx^2}{2u^2\cos^2\theta}$$

$$\text{or} \quad x = \frac{2u^2\sin\theta\,\cos\theta}{g} = \frac{u^2\sin 2\theta}{g}$$

But coordinates of M are $(r, 0)$. Putting $x = R$,

we have $\quad R = \frac{u^2 \sin 2\theta}{g}$.

Range is maximum when $\sin 2\theta = 1$.

i.e., $\quad 2\theta = \frac{\pi}{2}$ or 90° making $\theta = 45°$.

OR

(i) Given : Speed of plane = 720 km/hr. = 200 m/s. Speed of the shell = 600 m/s. Let θ be the angle from the vertical of the gun. Suppose that the shell strikes the plane at P, after time t when fired.

Therefore, the horizontal distance travelled by the shell in time t is equal to the distance travelled by the plane in the same time t.

$$\therefore \quad u\cos(90-\theta) \times t = 200 \times t$$

$$\text{or} \quad 600 \sin\theta = 200.$$

$$\text{or} \quad \sin\theta = \frac{200}{600} = \frac{1}{3} = \sin(19.5°)$$

or $\theta = 19.5°$ from the vertical.

In order to avoid being hit by the shell, the plane must fly at a height slightly greater than the maximum height attained by the shell.

$$H = \frac{u^2\sin^2\theta}{2g}$$

$$= \frac{(600)^2 \times \sin^2(90-\theta)}{2\times 10}$$

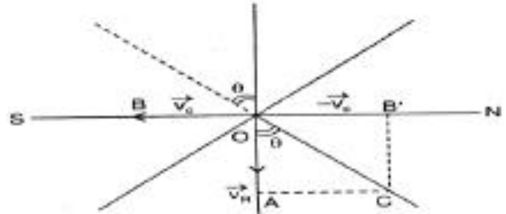
$$= \frac{360000 \times \cos^2\theta}{20}$$

$$= 18000 \times \left(\frac{\sqrt{8}}{3}\right)^2$$

$$= 16000 \text{ m.}$$

Thus, the plane should fly at a height slightly above 16 km.

(ii) Velocity of rain ($\vec{V}_R$) and the women $\vec{V}_C$ has been shown in figure by the vector $\vec{OA}$ and $\vec{OB}$ respectively.



Hence $\vec{V}_R$ = 30 m s⁻¹ (vertically downwards and V_C = 10 m s⁻¹ (from north to south)

In order to protect herself from the rain and women must hold her umbrella in the direction of relative velocity of the rain with respect to her. If $\vec{V}$ is the velocity of rain relative to the women, then

$$\vec{V} = \vec{V}_R + (-\vec{V}_C) = \vec{V}_R - \vec{V}_C$$

In the figure, the vector $\overline{OC}$ along the diagonal of the parallelogram OACB' represents $\vec{V} = V_R - V_C$. If θ is the angle that v makes with the vertical, then

$$\tan\theta = \frac{AC}{OA} = \frac{V_C}{V_R} = \frac{10}{30} = 0.3333$$

$\therefore \quad \theta = 18'26''$.(with vertical towards south)