1. Sol.

Let $z = -3\sqrt{2} + 3\sqrt{2}i$. Then

$$r = |z| = \sqrt{(-3\sqrt{2})^2 + (3\sqrt{2})^2} = 6$$

Let $\tan a = \left|\frac{\text{Im}(z)}{\text{Re}(z)}\right| = 1 \Rightarrow a = \pi/4$

Since the point representing z lies in the second quadrant. Therefore, the argument of z is given by

$$\theta = \pi - a = \pi - (\pi/4) = (3\pi/4).$$

So, the polar form of $z = -3\sqrt{2} + 3\sqrt{20}i$ is

$$z = r(\cos\theta + i\sin\theta) = 6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right).$$

2. Sol.

Let z = –4 + 0i. Then, $|z| = \sqrt{(-4)^2 + 0} = 4.$

Since the point (-4, 0) representing z = -4 + 0i lies on the negative side of real axis.

Therefore, principal argument of z is π.

3. Sol.

We have,

$$3x + 17 \le 2(1 - x)$$

$$\Rightarrow 3x + 17 \le 2 - 2x$$

$$\Rightarrow 3x + 2x \le 2 - 17$$

[Transposing – 2x to LHS and 17 to RHS]

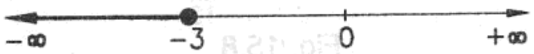
$$\Rightarrow 5x \le -15$$

$$\Rightarrow \frac{5x}{5} \le \frac{-15}{5}$$

$$\Rightarrow x \le -3$$

$$x \in (-\infty, -3)$$

Hence, the solution set of the given in equation is (-∞, -3], which can be graphed on real line as shown in Fig.



4. Sol.

We have,

$${}^9P_5 + 5 \cdot {}^9P_4 = {}^{10}P_r$$

$$\Rightarrow \frac{9!}{(9-5)!} + 5 \cdot \frac{9!}{(9-4)!} = \frac{10!}{(10-r)!}$$

$$\Rightarrow \frac{9!}{4!} + 5 \cdot \frac{9!}{5!} = \frac{10!}{(10-r)!}$$

$$\Rightarrow \frac{9!}{4!} + \frac{9!}{4!} = \frac{10!}{(10-r)!}$$

$$\Rightarrow 2 \times \frac{9!}{4!} = \frac{10!}{(10-r)!}$$

$$\Rightarrow \frac{5 \times 2 \times 9!}{5 \times 4!} = \frac{10!}{(10-r)!}$$

$$\Rightarrow \frac{10 \times 9!}{5!} = \frac{10!}{(10-r)!}$$

$$\Rightarrow \frac{10!}{5!} = \frac{10!}{(10-r)!}$$

$$\Rightarrow (10-r)! = 5! \Rightarrow 10 - r = 5 \Rightarrow r = 5$$

5. Sol.

There are 10 letters in the word 'LOGARITHMS'.

So, the number of 4 – letter word

= Number of arrangements of 10 letters, taken 4 at a time

= ${}^{10}P_4$ = 5040.

6. Sol.

Let there be n sides of the polygon. We know that the number of diagonals of n sided polygon is $\frac{n(n-3)}{2}$.

$$\therefore \frac{n(n-3)}{2} = 44 \Rightarrow n^2 - 3n - 88 = 0 \Rightarrow (n-11)(n+8) = 0 \Rightarrow$$

(∵ n > 0)

Hence, there are 11 sides of the polygon.

7. Sol.

(i) Let = y = x + x^2. Then,

$(1 + x + x^2)^3$

$$= (1+y)^3 = {}^3C_0 + {}^3C_1 y + {}^3C_2 y^2 + {}^3C_3 y^3$$

$$= 1 + 3y + 3y^2 + y^3 = 1 + 3(x + x^2) + 3(x + x^2)^2 + (x + x^2)^3$$

$$= 1 + 3(x + x^2) + 3(x^2 + 2x^3 + x^4) + \{{}^3C_0 x^3 (x^2)^0 + {}^3C_1 x^{3-1}(x^2)^1$$

$$= 1 + 3(x + x^2) + 3(x^2 + 2x^3 + x^4) + (x^3 + 3x^4 + 3x^5 + x^6)$$

$$= x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1$$

(ii) Let y = - x + x^2. Then,

$$(1 - x + x^2)^4$$

$$= (1+y)^4$$

$$= {}^4C_0 + {}^4C_1 y + {}^4C_2 y^2 + {}^4C_3 y^3 + {}^4C_4 y^4 = 1 + 4y + 6y^2 + 4y^3 + y^4$$

$$= 1 + 4(-x + x^2) + 6(-x + x^2)^2 + 4(-x + x^2)^3 + (-x + x^2)^4$$

$$= 1 - 4x(1-x) + 6x^2(1-x)^2 - 4x^3(1-x)^3 + x^4(1-x)^4$$

$$= 1 - 4x + 4x^2 + 6x^2(1 - 2x + x^2) - 4x^3(1 - 3x + 3x^2 - x^3) + x^4(1 - 4x + 6x^2 - 4x^3 + x^4)$$

$$= 1 - 4x + 4x^2 + 6x^2 - 12x^3 + 6x^4 - 4x^3 + 12x^4 - 12x^5 + 4x^6 + x^4 - 4x^5 + 6x^6 - 4x^7 + x^8$$

$$= 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$$

8. Sol.

We have, $\left(\frac{4x}{5}-\frac{5}{2x}\right)^9=\left\{\frac{4x}{5}+\left(-\frac{5}{2x}\right)\right\}^9$

$\therefore T_6=T_{5+1}={}^9C_5\left(\frac{4x}{5}\right)^{9-5}\left(-\frac{5}{2x}\right)^5$

[$\because T_{r+1}={}^nC_r x^{n-r}a^r$]

$\Rightarrow T_6={}^9C_5\left(\frac{4x}{5}\right)^4(-1)^5\left(\frac{5}{2x}\right)^5=-{}^9C_4\left(\frac{4x}{5}\right)^4\left(\frac{5}{2x}\right)^5$

[$\because {}^9C_5={}^9C_4$]

$\Rightarrow T_6=-\frac{9\times8\times7\times6}{4\times3\times2\times1}\left(\frac{2^8}{5^4}\cdot x^4\right)\left(\frac{5^5}{2^5\cdot x^5}\right)=-\frac{5040}{x}$

9. Sol.

We known that in a A.P. the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of first and last term i.e. $a_1+a_n=a_2+a_{n-1}=a_3+a_{n-2}=\ldots$ So, if an A.P. consists of 24 terms, then $a_1+a_{24}=a_5+a_{20}=a_{10}+a_{15}$.

Now, $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225$

$\Rightarrow(a_1+a_{24})+(a_5+a_{20})+(a_{10}+a_{15})=225$

$\Rightarrow 3(a_1+a_{24})=225\Rightarrow a_1+a_{24}=\frac{225}{3}=75$ ...(i)

$\therefore S_{24}=\frac{24}{2}(a_1+a_{24})\quad\left[\text{Using}S_n\frac{n}{2}(a_1+a_n)\right]$

$\Rightarrow S_{24}=12(75)=900$ [Using (i)]

10. Sol.

Let A be the first term and R the common ration of the given G.P. Then,

$a=pth\,term\Rightarrow a=AR^{p-1}\Rightarrow\log a=\log A+(p-1)\log R$ ... (i)

$b=qth\,term\Rightarrow b=AR^{q-1}\Rightarrow\log b=\log A+(q-1)\log R$ ... (ii)

$c=rth\,term\Rightarrow c=AR^{r-1}\Rightarrow\log c=\log A+(r-1)\log R$ ... (iii)

Now, (q - r) log a + (r - p) log b + (p - q) log c

$=(q-r)\{\log A+(p-1)\log R\}+(r-p)\{\log A+(q-1)\log R\}+(p-q)\{\log A+(r-1)\log R\}$ [Using (i), (ii) and (iii)]

$=\log A\{(q-r)+(r-p)+(p-q)\}+\log R\{(P-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)\}$

$=\log A.0+\log R\{p(q-r)+q(r-p)+r(p-q)-(q-r)-(r-p)-(p-q)\}$

$=\log A.0+\log R.0=0$

11. Sol.

$\frac{a^{n+1}+b^{n+1}}{a^n+b^b}$ is the G.M. between a and b

$\Leftrightarrow\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\sqrt{ab}$

$\Rightarrow a^{n+1}+b^{n+1}=a^{(n+\frac{1}{2})}b^{1/2}+a^{1/2}b^{(n+\frac{1}{2})}$

$\Leftrightarrow a^{n+1}-a^{(n+\frac{1}{2})}b^{1/2}=a^{1/2}b^{(n+\frac{1}{2})}-b^{n+1}$

$\Leftrightarrow a^{(n+\frac{1}{2})}(a^{1/2}-b^{1/2})=b^{(n+\frac{1}{2})}(a^{1/2}-b^{1/2})$

$\Leftrightarrow a^{(n+\frac{1}{2})}=b^{(n+\frac{1}{2})}$ [$\because a^{1/2}-b^{1/2}\neq0$, as $a\neq b$]

$\Leftrightarrow\left(\frac{a}{b}\right)^{(n+\frac{1}{2})}=1$

$\Leftrightarrow\left(\frac{a}{b}\right)^{(n+\frac{1}{2})}=\left(\frac{a}{b}\right)^0\Leftrightarrow n+\frac{1}{2}=0\Leftrightarrow n=-\frac{1}{2}$

12. Sol.

We have,

$S_1=\sum_{k=1}^{n}k=1+2+\ldots+n=\frac{n(n+1)}{2}$

$S_2=\sum_{k=1}^{n}k^2=1^2+2^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$

$S_3=\sum_{k=1}^{n}k^3=1^3+2^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$

$\therefore 9S_2^2=9\left(\frac{n(n+1)(2n+1)}{6}\right)^2=\frac{9}{36}\{n+(n+1)(2n+1)\}^2=\frac{1}{4}\{n(n+1)(2n+1)\}^2$

and,

$S_3(1+8S_1)=\left\{\frac{n(n+1)}{2}\right\}^2\left\{1+8.\frac{n(n+1)}{2}\right\}=\left\{\frac{n(n+1)}{2}\right\}^2\{4n^2+4n+1\}$

$=\frac{n^2(n+1)^2(2n+1)^2}{4}=\frac{1}{4}\{n(n+1)(2n+1)\}^2$

Hence, $9S_2^2=S_3(1+8S_1)$.

13. Sol.

The equation of the line joining the points (2, 3) and (4, 1) is

$y-3=\frac{1-3}{4-2}(x-2)$

$\Rightarrow y-3=-x+2\Rightarrow x+y-5=0$ …. (i)

Suppose the line joining (2, 3) and (4, 1) divides the segment joining (1, 2) and (4, 3) at point P in the ration λ : 1. Then the coordinates of P are

$$\left(\frac{4\lambda+1}{\lambda+1},\frac{3\lambda+2}{\lambda+1}\right)$$

Clearly P lies on (i),

$$\therefore \frac{4\lambda+1}{\lambda+1}+\frac{3\lambda+2}{\lambda+1}-5=0 \Rightarrow \lambda=1.$$

Hence, the required ration is λ : 1 i.e. 1 : 1.

14. Sol.

The equation of line passing through P (3, 4) and making an angle $\frac{\pi}{6}$ with x-axis is

$$\frac{x-3}{\cos\frac{\pi}{6}}=\frac{y-4}{\sin\frac{\pi}{6}}=r \text{ or, } \frac{x-3}{\frac{\sqrt{3}}{2}}=\frac{y-4}{\frac{1}{2}}=r$$

Where r represents the distance of any point on this line from the given point P (3, 4).

The coordinates of any point Q on this line are

$$\left(3+\frac{\sqrt{3}}{2}r, 4+\frac{r}{2}\right)$$

If Q lies on 12x + 5y + 10 = 0, then

$$12\left(3+r\frac{\sqrt{3}}{2}\right)+5\left(4+\frac{r}{2}\right)+10=0$$

$$\Rightarrow r=\frac{-132}{12\sqrt{3}+5}$$

Hence, length $PQ=\frac{132}{12\sqrt{3}+5}$.

15. Sol.

The equation of any line parallel to the line 3x – 2y + 5 = 0 is

3x – 2y + λ = 0 ... (i)

This passes through (5, -6).

∴ 3 × 5 – 2 × –6 + λ = 0 ⇒ λ = –27.

Putting λ = - 27 in (i), we get 3x – 2y – 27 =0, which is the required equation.

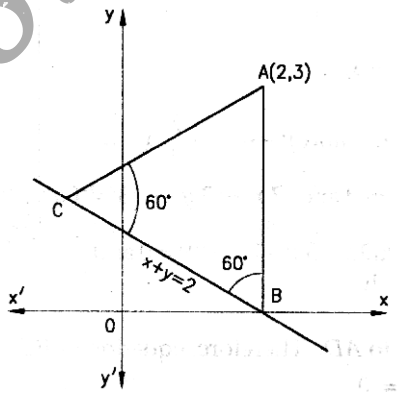
<u>ALITER</u>

The slope of the given line is 3/2. Therefore the slope of the required line is also 3/2. Since the required line passes through (5, -6) so its equation is

$$y+6=\frac{3}{2}(x-5) \Rightarrow 3x-2y-27=0$$

16. Sol.

Let A (2, 3) be one vertex and x + y = 2 be the opposite side of an equilateral triangle. Clearly remaining two sides pass through the point A (2, 3) and make and angle of 60° with x + y = 2.



Let m be the slope of x + y = 2. Then m = -1.

So, the equations of the other two sides are

$$y-3=\frac{-1-\tan 60^\circ}{1-\tan 60^\circ}(x-2) \text{ and}$$

$$y-3=\frac{-1+\tan 60^\circ}{1+\tan 60^\circ}(x-2)$$

$$\Rightarrow y-3=\frac{-(1+\sqrt{3})}{1-\sqrt{3}}(x-2) \text{ and}$$

$$y-3=\frac{\sqrt{3}-1}{\sqrt{3}+1}(x-2)$$

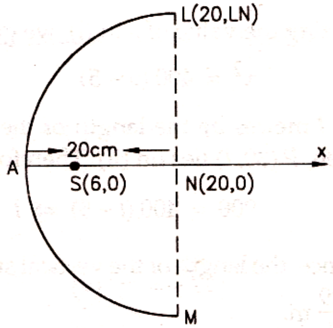
$$\Rightarrow y-3=(2+\sqrt{3})(x-2) \text{ and}$$

$$(y-3)(2+\sqrt{3})=x-2$$

$$\Rightarrow (2+\sqrt{3})x-y=1+2\sqrt{3} \text{ and}$$

$$(2-\sqrt{3})x-y=1-2\sqrt{3}.$$

17. Sol.



Let the axis of the mirror be along the positive direction of x-axis and the vertex A be the origin.

Since the focus is at a distance of 6 cm from the vertex. Then, the coordinates of the focus are (6, 0). Therefore, the equation of the parabolic section is

$y^2 = 24x$ [Putting a = 6 in $y^2 = 4ax$]

Since L (20, LN) lies on this parabola. Therefore,

$$LN^2 = 24 \times 20 \Rightarrow LN = 4\sqrt{30}$$

$$\therefore LM = 2LN = 8\sqrt{30} cm.$$

18. **Sol.**

We have,

$$x^2 + 4y^2 + 2x + 16y + 13 = 0$$

$$\Rightarrow (x^2 + 2x + 1) + 4(y^2 + 4y + 4) = 4$$

$$\Rightarrow (x+1)^2 + 4(y+2)^2 = 4$$

$$\Rightarrow \frac{(x+1)^2}{2^2} + \frac{(y+2)^2}{1^2} = 1 \quad .... (i)$$

Shifting the origin at (-1, -2) without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have

x = X – 1 and y = Y – 2 .... (ii)

Using these relations, equation (i) reduced to

$$\frac{X^2}{2^2} + \frac{Y^2}{1^2} = 1$$

This is of the form $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$, where a = 2 and b = 1.

Thus, the given equation represents an ellipse.

Clearly, a > b. So, the given equation represents an ellipse whose major and minor axes are along X and Y axes respectively.

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

Vertices: The vertices of the ellipse with respect to the new axes are (X = ±a, Y = 0) i.e. (X = ± 2, Y = 0). So the vertices with respect to the old axes are given by

(± 2 -1, -2) i.e. (-3, -2) and [Using (ii)]

Foci: The coordinates of the foci with respect to the new axes are given by (X = ±ae, Y = 0) i.e. $(X = \pm\sqrt{3}, Y = 0)$.

So, the coordinates of foci with respect to the old axes are given by

$(\pm\sqrt{3} - 1, -2)$ [Putting $(X = \pm\sqrt{3}, Y = 0)$ in (ii)]

Directrices: The equations of the directrices with respect to the new axes are $X = \pm\frac{a}{e}$ i.e.

$$X = \pm\frac{4}{\sqrt{3}}.$$

So, the equations of the directrices with respect to the old axes are

$$x = \pm\frac{4}{\sqrt{3}} - 1 \text{ i.e. } x = \frac{4}{\sqrt{3}} - 1 \text{ and } x = -\frac{4}{\sqrt{3}} - 1 \left[\text{Putting } X = \pm\frac{4}{\sqrt{3}} \text{ in } (ii)\right]$$

Length of the latus-rectum: The length of the latusrectum $= \frac{2b^2}{a} = \frac{2}{2} = 1$

Equations of Latus-recta: The equations of the latusrecta with respect to the new axes are

X = ±ae i.e. X = $\pm\sqrt{3}$

So, the equations of the latusrecta with respect to the old axes are

$x = \pm\sqrt{3} - 1$ [Putting $X = \pm\sqrt{3}$ in (ii)]

i.e. $x = \sqrt{3} - 1$ and $x = -\sqrt{3} - 1$.